**Optimization Project**

Your task is to analyze the capacity expansion, production, and distribution for a new product at XYZ Manufacturing, and write a report on your conclusions and recommendations. When deciding what information to include in your report, keep in mind that the report is intended for management personnel. That is, management would tend to be much more interested in the system requirements and analysis than in the details of the mathematical model.

A complete description of the problem follows.

**Problem Description**

XYZ Manufacturing plans to distribute a new product, the Flugel and would like to do some long-range planning regarding its production and distribution. There are 8 Retail Centers where the product will be sold and the Forecasting Department has estimated that the year-one demand for Flugels will be as follows:

|  |  |
| --- | --- |
| Retail Center | Year-1 Demand |
| 1 | 1000 |
| 2 | 1200 |
| 3 | 1800 |
| 4 | 1200 |
| 5 | 1000 |
| 6 | 1400 |
| 7 | 1600 |
| 8 | 1000 |

Forecasting has also estimated that demand at Retail Centers 1, 2, 4, 5, and 8 will grow yearly at a rate **of 20% of the year-1 demand. For example, the demand at Retail Center 1 in year *t* is estimated to be *1000 + 0.20(1000)(t – 1)*. The corresponding percentage for Retail Centers 3, 6, and 7 is 25%.**

The company has several existing plants and has a choice of building (and then operating) a new production line at the plants with the specifications and estimated year-1 costs shown in the following table. In addition to construction costs and yearly operating costs, XYZ may shut down a line that is operating or reopen a line that has been previously shut down. You can envision a line that is shut down as being converted for other plant purposes. Similarly, a line that is reopened is being converted back to Flugel production. For simplicity, assume that a line may be reopened at the beginning of **each year** or shut down at the end of each year. Also assume that in the initial construction year, a line incurs construction costs as well as start-up (i.e., reopening) costs. However, if the line has already been constructed and is then shut down at the end of some year, the plant only incurs the reopening cost to begin production again. Of course, operating costs are only realized if production actually occurs in a given year. And, clearly, a production line cannot be allowed to remain idle in a given year.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Plant | Capacity (units) | Construction Cost (1000's of $) | Annual Operating Cost (1000's of $) | Reopening Cost (1000's of $) | Shutdown Cost (1000s of $) |
| 1 | 16000 | 2000 | 420 | 190 | 170 |
| 2 | 12000 | 1600 | 380 | 150 | 120 |
| 3 | 14000 | 1800 | 460 | 160 | 130 |
| 4 | 10000 | 900 | 280 | 100 | 80 |
| 5 | 13000 | 1500 | 340 | 130 | 110 |

Flugels will be stored and distributed using XYZ’s existing Warehouses. **There is no specific per unit charge for storage at a Warehouse**. However, since stored items require considerable capital investment XYZ would like to limit the number of units stored. Over the course of a year, obviously the inventory level fluctuates as items flow into and out of the warehouses. **To simplify modeling, XYZ has decided to model an average inventory level based on the inventory level at the beginning of the year and at the end of the year.**  The average inventory level for the year will be the average of these two inventory levels. XYZ has determined, based on a simple average cost model, that they can keep the capital investment within reasonable bounds if they maintain the average inventory in any year to be no more than 4000 items (among all Warehouses).

The existing warehouse system handles many other products and services, and XYZ estimates that the maximum that any warehouse can realistically handle is an average of 1000 Flugels per month (i.e., 12000 per year). Since some units may be stored in a warehouse, both the flow into a warehouse and the flow out of a warehouse should not exceed an average of 1000 units per month.

In addition to the operating expenses (which include overhead items such as electrical power, salaries, etc.), the production of Flugels requires two key resources. Each Flugel requires 4.7 pounds of alloy, and a given plant can acquire a maximum of 60000 pounds of alloy per year. The year-1 cost for one pound of alloy is estimated to be 0.02 (in 1000’s of $). Also, each Flugel requires 3 **Widget subassemblies**. The year-1 cost of each Widget subassembly is 0.15 (in 1000’s of $), however, at a given plant, it is estimated that the cost will drop to 0.12 (in 1000’s of $) after the first 9000 Widget subassemblies have been purchased in a particular year. Each plant can purchase as many **Widget subassemblies** as needed, and it is assumed that both resources will be supplied on a just-in-time basis.

The estimated year-1 cost of shipping **a Flugel** from Plant *i* to Warehouse *j* is given in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Plant | Warehouse 1 (1000's of $) | Warehouse 2 (1000's of $) | Warehouse 3 (1000's of $) | Warehouse 4 (1000's of $) |
| 1 | 0.12 | 0.13 | 0.08 | 0.05 |
| 2 | 0.1 | 0.03 | 0.1 | 0.09 |
| 3 | 0.05 | 0.07 | 0.06 | 0.03 |
| 4 | 0.06 | 0.03 | 0.07 | 0.07 |
| 5 | 0.06 | 0.02 | 0.04 | 0.08 |

Finally, the estimated year-1 **unit** shipping cost from each Warehouse to each Retail Center is given in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Warehouse | Retail 1 (1000's of $) | Retail 2 (1000's of $) | Retail 3 (1000's of $) | Retail 4 (1000's of $) |
| 1 | 0.09 | 0.1 | 0.06 | 0.05 |
| 2 | 0.05 | 0.07 | 0.12 | 0.04 |
| 3 | 0.06 | 0.09 | 0.07 | 0.09 |
| 4 | 0.07 | 0.08 | 0.09 | 0.06 |
|  |  |  |  |  |
|  |  |  |  |  |
| Warehouse | Retail 5 (1000's of $) | Retail 6 (1000's of $) | Retail 7 (1000's of $) | Retail 8 (1000's of $) |
| 1 | 0.08 | 0.09 | 0.02 | 0.12 |
| 2 | 0.03 | 0.09 | 0.03 | 0.08 |
| 3 | 0.09 | 0.04 | 0.11 | 0.07 |
| 4 | 0.1 | 0.07 | 0.06 | 0.09 |

Costs are expected to rise at the rate of 3% per year. That is, if *c* is a particular cost in year 1, then the cost in year *t* is estimated to be *c(1 + 0.03)t-1*.

Determine the strategy that **minimizes** the total cost of meeting the expected demand over the next 10 years.